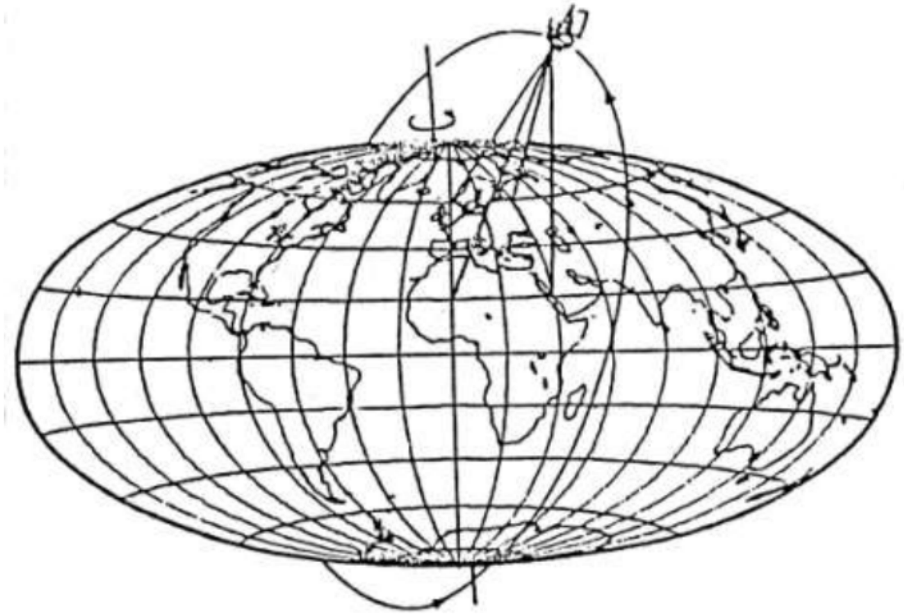


G E O D E S Y

Part 2

Prof. Dr. Ahmed A. Shaker

Prof. Of Surveying and Geodesy

Shoubra Faculty of Engineering

Chapter 5

Reference systems

5-1 Introduction

It is known that by means of certain mathematical operations we can transfer our physical observations into geodetic information of position, azimuth, elevation, distance, or size and shape of the earth.

A coordinate system is necessary for all our calculations , whether as an intermediate step or end result. Such a reference system may take many forms of which sometimes one, and sometimes another may be the most convenient. Some of these systems, which are of special importance in geodesy, may be described briefly as follows:

5-2 Natural Coordinate System Φ, Λ, H

Astronomical observations for latitude, longitude, and azimuths are measured with reference to the direction of gravity at the point of observation. In the natural coordinate system the position of any point on the earth's surface can be fixed by observing its astronomic latitude, longitude, and its orthometric height, figure (5-1).

1) Astronomical Latitude Φ: is the angle between the equatorial plane and the direction of the vertical at the point of observation.
2) Astronomical Longitude Λ: is the angle between the meridian plane of the observation point and the meridian plane of Greenwich.
3) Orthometric Height H: is the height of a point above mean sea level. It is measured along the curved plumb line and obtained from spirit levelling and gravity observations.

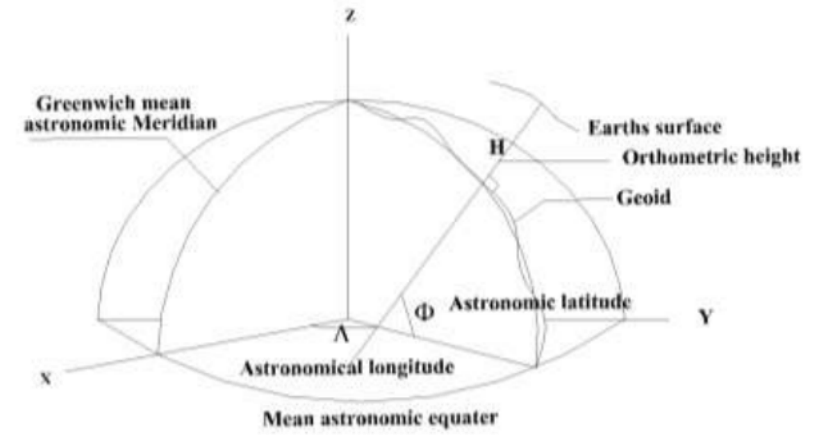


FIG 5-1

Consequently, the quantities Φ, Λ, and H define the position of the observer with respect to the geoid & the mean rotational axis of the earth.

5-3 Geodetic Coordination System ϕ, λ, h

Since the deviations of the geoid from the reference ellipsoid are small and can be computed, it is convenient to add small reductions to the observed coordinate so that, values refer to an ellipsoid can be established, which are called geodetic coordinates, figure (5-2).

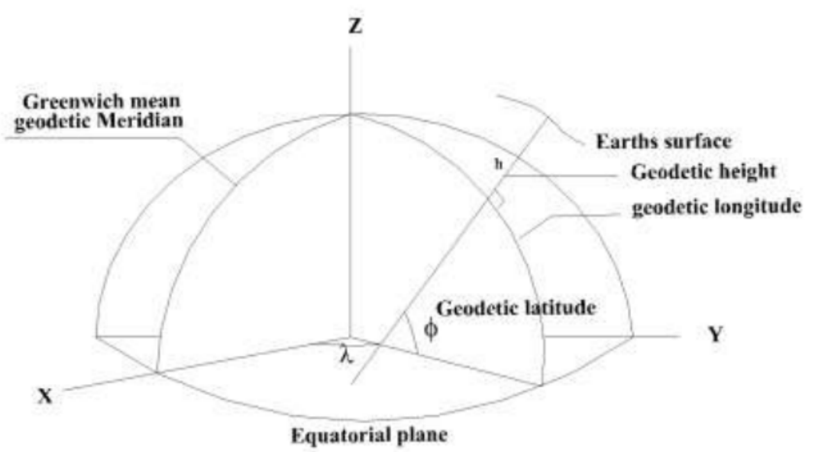


FIG 5-2

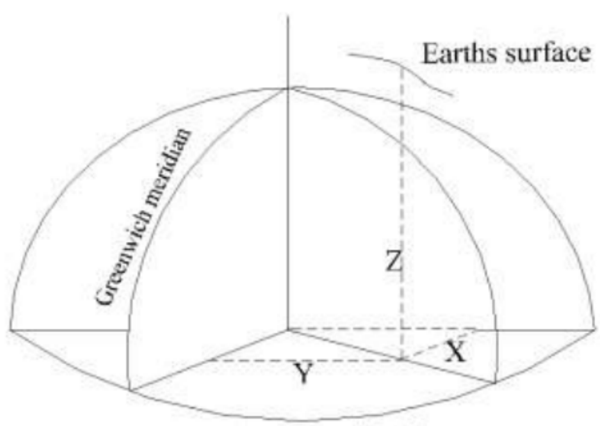
1) Geodetic Latitude ϕ: is the angle between the ellipsoidal normal of the observers projected position on the geoid and the perpendicular to the mean rotation axis of the earth.
2) Geodetic Longitude λ: is the angle between the same ellipsoidal normal and Greenwich meridian plane.
3) Geodetic Height h: is the height of the observer above the reference ellipsoid, measured along the ellipsoidal normal.

The geodetic coordinates are determined from *Triangulation* or *Trilateration* observed on the earth surface, reduced to the ellipsoid.

They could also be obtained directly from the astronomic coordinates reduced to the used reference ellipsoid.

5- 4 Rectangular Coordinate System X, Y, Z.

Generally, it is convenient to take the X-axis parallel to the meridian of Greenwich; the Y-axis has the longitude of 90° east of Greenwich, and the Z-axis parallel to the CIO (conventional international origin of polar motion). Ideally the origin of the rectangular coordinates system should be at the earth's center of gravity; the system is known as "Average Terrestrial Coordinate System". When the origin is at the geometric center of the ellipsoid, and not in the (C.G.) of the earrth, it is known as "Geodetic Coordinate System" figure (5-3).



Geodetic equatorial plane

FIG 5-3

5-5 Local Coordinate System (Horizon System) U, V, W.

In this system the coordinates U, V, W are expressed as functions of the observed azimuth A, zenith distances Z & spatial distance S. figure (5-4)

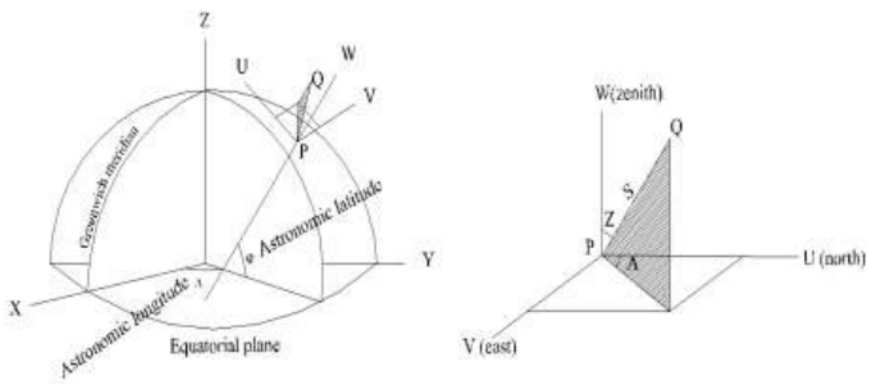


FIG 5-4

illustrates the quantities of this system. The origin is considered to be at the observation station P. the positive U-axis points N-ward, the positive V-axis points eastward and the positive W-axis coincides with the outward direction of the plumb line.

The coordinate equations of an object Q sighted in this system may be written simply by referring to the fig.

$$U = S\sin Z\cos A$$

$$V = S\sin Z\sin A \qquad (5\text{-}1)$$

$$W = S\cos Z$$

5- 6 Relations between different Reference Systems

There are certain mathematical relations connecting the previous coordinate systems. These relations are among the basic equations in geodesy. Some of these relations will be considered in the next subsections.

5-6-1 Relation between Astronomic and Geodetic Coordinates

Since the astronomical system depends on the direction of the vertical "actual gravity field", while the geodetic system depends on the direction of the ellipsoidal normal "normal gravity field", then the relation between both systems depends mainly on the difference between the two directions. The total difference between the two directions is the well-known deflection of the vertical θ. It has two components, a north-south component ξ and an east-west component η. We can read from figure (5-5) the following:

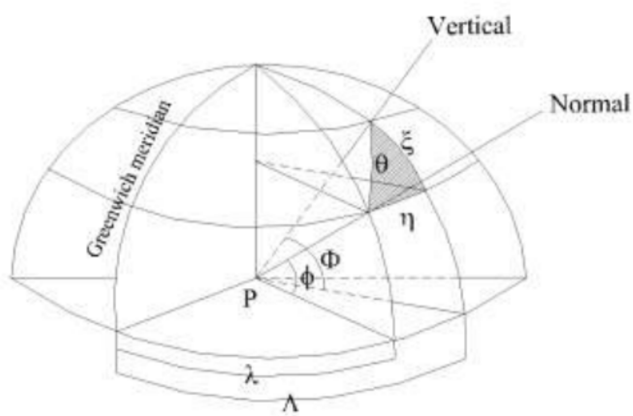


FIG 5-5

$$\xi = \Phi - \phi \qquad (5\text{-}2a)$$

$$\eta = (\Lambda - \lambda)\cos\phi \qquad (5\text{-}2b)$$

$$\theta = (\xi^2 + \eta^2)^{0.5} \qquad (5\text{-}2c)$$

According to Helmert's projection, which neglects the curvature of the plumb line, a point P on the earth's surface is directly projected onto the ellipsoid by means of straight ellipsoidal normal, point P_1. Then the ellipsoidal height is given by

$$h = H + N \qquad (5\text{-}3)$$

In pursuing this relationship, it is important to remember Pizzetti's projection, figure (5-6). In this projection the same point P is projected along the curved plumb line onto

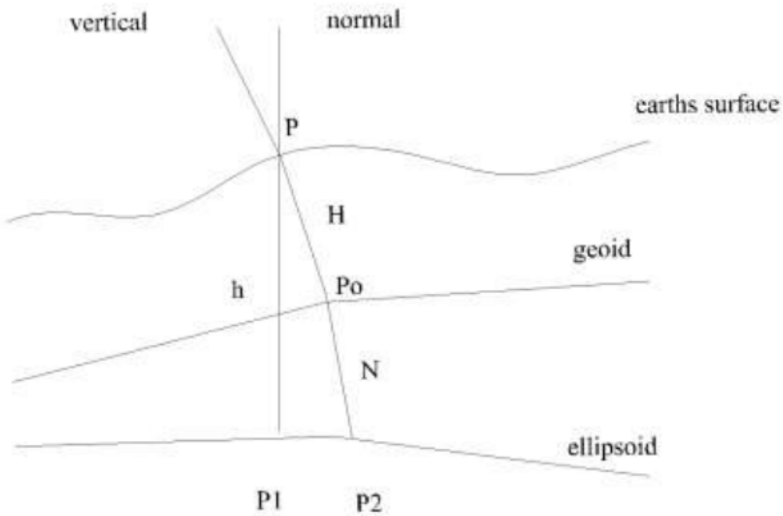


FIG 5-6

the geoid, point Po, and then projected onto the ellipsoid, point P2. The practical difference between the two projections is small, and within a fraction of millimeters.

5-6-2 Relation between Rectangular and Curvilinear Geodetic Coordinates:

The coordinate transformation between the curvilinear geodetic coordinates and the Cartesian coordinates may be expressed symbolically by

$$(\phi, \lambda, h) \xleftrightarrow[(a,f)]{} (X, Y, Z)$$

From figure (5-7) the relation between the two systems can be written as follows;

$$R_P = (N + h)\cos\phi \qquad (5\text{-}4)$$

$$X = R_P \cos\lambda$$

$$Y = R_P \sin\lambda$$

where,

$N =$ Radius of curvature in the prime vertical

$$N = \frac{a}{(1 - e^2 \sin^2 \phi)^{0.5}}$$

$h =$ Ellipsoidal height

Also, from same figure, we can read

$$Z = Z_1 - K$$

where,

$Z_1 = (N + h)\sin\phi$

$K = N \cdot e^2 \sin\phi$

Combining the latter expression with the formers, we get:

$$X = (N + h)\cos\phi\cos\lambda \qquad (5\text{-}5a)$$

$$Y = (N + h)\cos\phi\sin\lambda \qquad (5\text{-}5b)$$

$$Z = (N(1 - e^2) + h)\sin\phi \qquad (5\text{-}5c)$$

These equations are the basic transformation formulas between the geodetic coordinates ϕ, λ, h and the rectangular coordinates X, Y, Z of a point outside the ellipsoid. The origin of the rectangular coordinate system is the center of the ellipsoid, and the Z-axis is its axis of rotation; the X-axis has the Greenwich 0° longitude and the Y-axis has longitude 90° east of Greenwich (i.e., $\lambda = +90^\circ$).

Inverse Procedure:

The computation of ϕ, λ, h from given X, Y, Z is more complicated because the three equations have four unknowns, N including ϕ. Accordingly, the computation could be done iteratively in addition to the direct solution. Many solutions, through iteration, were given for this problem, for example; *HIRVONEN & MORITZ 1963 BARTELME & MEISEL 1975, RAPP* and *KRAUSS 1976*. Also a non-iterative solution was given by *SUENKEL 1976*.

For the iterative solution we shall follow *(Hirvonen & Moritz 1963)*. Now from figure (5-7) we find

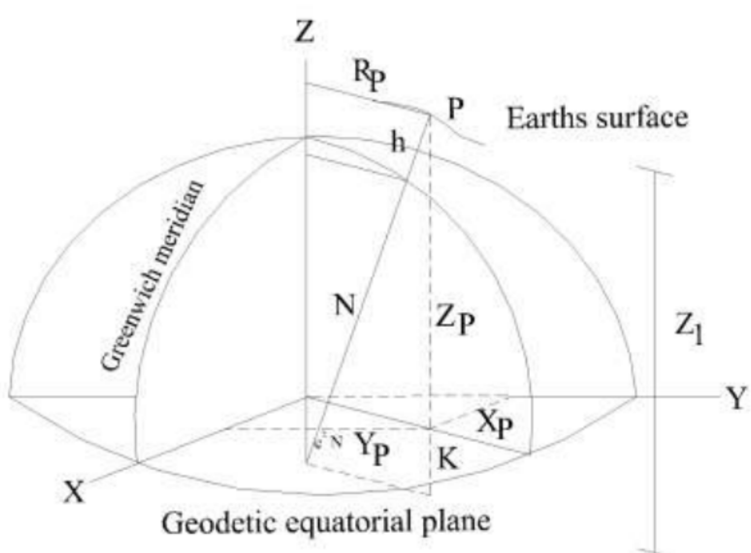


FIG 5-7

$R_P = (X^2 + Y^2)^{0.5} = (N + h)\cos\phi$

hence

$$h = \frac{R_P}{\cos\phi} - N \tag{5-6}$$

Equation (5-5) may be transformed into

$$Z = (N - \frac{a^2 - b^2}{a^2} N + h)\sin\phi$$

$$Z = (N + h - e^2 N)\sin\phi$$

where

$$e^2 = (a^2 - b^2) / a^2$$

Dividing this equation by the above expression for R_P we get

$$\frac{Z}{R_P} = (1 - e^2 \frac{N}{N+h})\tan\phi$$

so that

$$\tan\phi = \frac{Z}{R_P}(1 - e^2 \frac{N}{N+h})^{-1} \tag{5-7}$$

Given X, Y, Z and hence, R_P equations (5-6) and (5-7) may be solved iteratively for h and ϕ.

As a first approximation, we set $h = 0$ in (5-7), obtaining

Using ϕ_1, we compute an approximate value N_1 by means of

$$N_1 = \frac{a}{(1 - e^2 \sin^2 \phi_1)^{0.5}} \tag{5-8}$$

and introduce this value of N_1 in equation (5-6) to get an approximate value h_1.

$$h_1 = \frac{R_P}{\cos\phi_1} - N_1$$

Now, as a second approximation, we set $h = h_1$ in (5-7) obtaining

$$\tan\phi_2 = \frac{Z}{R_p}(1 - e^2 \frac{N_1}{N_1 + h_1})^{-1}$$

Using ϕ_2, improved values for N & h are found, etc. This procedure is repeated until the values of ϕ & h remain practically constant. The third value λ can be easily calculated from

$$\tan\lambda = Y/X$$

$$\tan\phi_1 = \frac{Z}{R_p}(1-e^2)^{-1} \qquad (5\text{-}9)$$

5-6-3 Relation between Horizon and Rectangular System

Since all the observations in geodesy, mainly horizontal, vertical angles, and spatial distances, are made with respect to the direction of the vertical at the observation station. Then it is important to find out the relations connecting these observable quantities of these two systems. Figure (5-8) illustrates the quantities of these two systems, where point P represents the occupied station, Q is the observed objects, and ***pq*** is the horizontal projection of the spatial distance S onto the horizon plane ***Psqn*** of the local system U,V,W. Then by equation (5-1) we can compute u,v,w of any point Q from station P. The plane through points M,O,q' is parallel to the equatorial plane of the X,Y,Z system. The projection of this horizon plane on $MOq'R$ plane is given as follows

$$MO = NO + MN$$

$$MO = w\cos\Phi - u\sin\Phi \qquad (5\text{-}10)$$

Likewise

$$\Delta X = Ma - Ob = MO\cos\Lambda - v\sin\Lambda$$
$$\Delta Y = aO - bq = MO\sin\Lambda + v\cos\Lambda \qquad (5\text{-}11)$$
$$\Delta Z = -PM + qQ = u\cos\Phi + w\sin\Phi$$

Then, the final form are achieved by combining these relations together as follows

$$\begin{vmatrix}\Delta X\\ \Delta Y\\ \Delta Z\end{vmatrix} = \begin{vmatrix}-\sin\Phi\cos\Lambda & -\sin\Lambda & \cos\Phi\cos\Lambda\\ -\sin\Phi\sin\Lambda & \cos\Lambda & \cos\Phi\sin\Lambda\\ \cos\Phi & 0 & \sin\Phi\end{vmatrix} \cdot \begin{vmatrix}u\\ v\\ w\end{vmatrix}$$

Or in matrix notation $\Rightarrow X = R^T \cdot u$

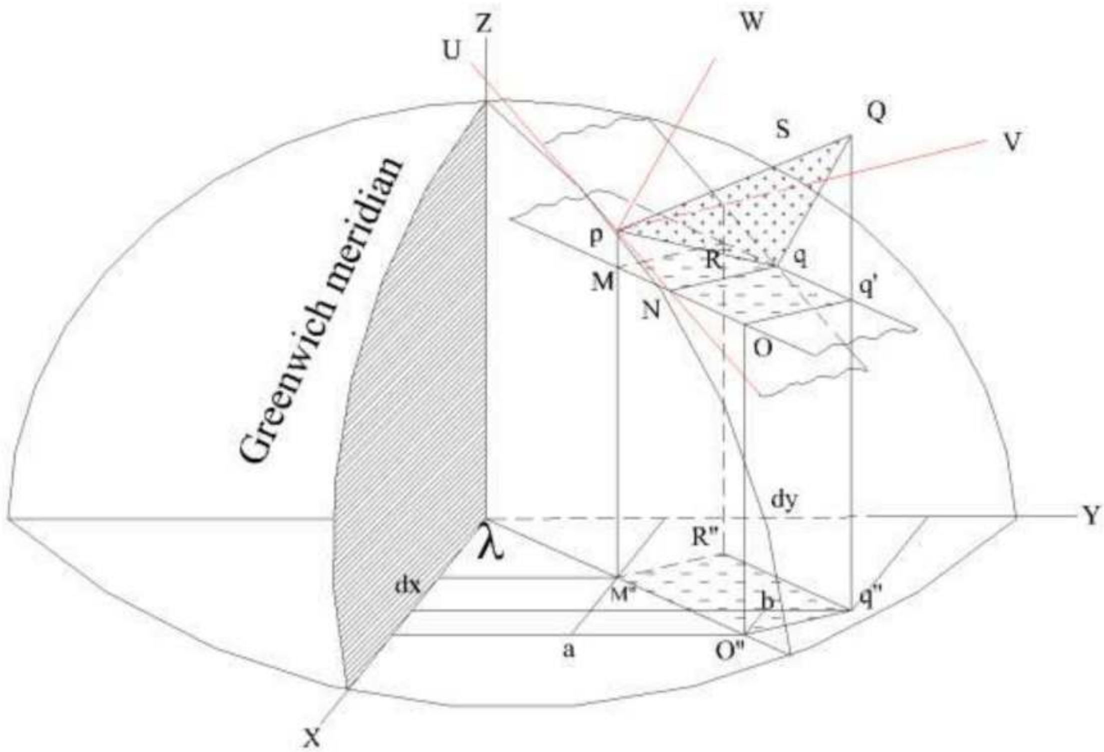


FIG 5-8